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## CASIMIR EFFECT IN TOROIDALLY COMPACTIFIED DE SITTER SPACETIME

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Vacuum energy density and stresses are investigated for a scalar field with general curvature coupling parameter in  $(D + 1)$ -dimensional de Sitter spacetime with an arbitrary number of toroidally compactified spatial dimensions. The corresponding expectation values are presented in the form of the sum of the vacuum expectation values in uncompactified dS spacetime and the part induced by the non-trivial topology. In the early stages of the cosmological evolution the topological parts dominate. In this limit the behavior of the Casimir densities does not depend on the curvature coupling parameter and coincides with that for a conformally coupled massless field. At late stages of the cosmological expansion the expectation values are dominated by the part corresponding to uncompactified dS spacetime. The vanishing of the topological parts is monotonic or oscillatory in dependence of the mass and the curvature coupling parameter of the field.

*Keywords:* Casimir effect; de Sitter spacetime; non-trivial topology.

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### 1. Introduction

Many of high-energy theories of fundamental physics are formulated in higher-dimensional spacetimes. In particular, the idea of extra dimensions has been extensively used in supergravity and superstring theories. It is commonly assumed that the extra dimensions are compactified. From an inflationary point of view universes with compact spatial dimensions, under certain conditions, should be considered a rule rather than an exception<sup>1</sup>. The models of a compact universe with non-trivial topology may play an important role by providing proper initial conditions for inflation. As it was argued in Refs. 2, there are many reasons to expect that in string theory the most natural topology for the universe is that of a flat compact three-manifold.

In topologically non-trivial spaces the periodicity conditions imposed on possible field configurations change the spectrum of the vacuum fluctuations and lead to the

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Casimir-type contributions to the vacuum expectation values of physical observables (for the topological Casimir effect and its role in cosmology see Refs. 3, 4). In the Kaluza-Klein-type models the Casimir effect has been used as a stabilization mechanism for moduli fields which parametrize the size and the shape of the extra dimensions. The Casimir energy can also serve as a model for dark energy needed for the explanation of the present accelerated expansion of the universe (see Refs. 5 and references therein). In the present talk, based on Refs. 6, 7, we describe the effects of the toroidal compactification of spatial dimensions in dS spacetime on the properties of quantum vacuum for a scalar field with general curvature coupling parameter. The one-loop quantum effects for a fermionic field on background of dS spacetime with spatial topology  $\mathbb{R}^p \times (\mathbb{S}^1)^q$  are studied in Refs. 8, 9.

The paper is organized as follows. In the next section we describe the background geometry and present the complete set of eigenfunctions. In section 3 these eigenfunctions are used to evaluate the mode-sum for the vacuum expectation value of the energy-momentum tensor. The main results are summarized in section 4.

## 2. Background geometry and the eigenfunctions

As a background geometry we consider  $(D + 1)$ -dimensional de Sitter spacetime ( $\text{dS}_{D+1}$ ) generated by a positive cosmological constant  $\Lambda$ . In planar (inflationary) coordinates the corresponding line element has the form

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^D (dz^l)^2, \quad (1)$$

with the parameter  $\alpha^2 = D(D-1)/(2\Lambda)$ . We will assume that the spatial coordinate  $z^l$ ,  $l = p+1, \dots, D$ , is compactified to  $\mathbb{S}^1$  of the length  $L_l$ :  $0 \leq z^l \leq L_l$ , and for the other coordinates we have  $-\infty < z^l < +\infty$ ,  $l = 1, \dots, p$ . Hence, we consider the spatial topology  $\mathbb{R}^p \times (\mathbb{S}^1)^q$ , where  $q = D - p$ .

This paper is concerned with the scalar vacuum densities induced by the non-trivial spatial topology. We will consider a free scalar field with curvature coupling parameter  $\xi$ . The corresponding field equation has the form

$$(\nabla_l \nabla^l + m^2 + \xi R) \varphi = 0, \quad (2)$$

where  $R = D(D+1)/\alpha^2$  is the Ricci scalar for  $\text{dS}_{D+1}$  and  $\xi$  is the curvature coupling parameter. Let  $\mathbf{z}_p = (z^1, \dots, z^p)$  and  $\mathbf{z}_q = (z^{p+1}, \dots, z^D)$  be the position vectors along the uncompactified and compactified dimensions respectively. We have the following boundary condition along the compactified dimensions

$$\varphi(t, \mathbf{z}_p, \mathbf{z}_q + L_l \mathbf{e}_l) = \pm \varphi(t, \mathbf{z}_p, \mathbf{z}_q), \quad (3)$$

where  $l = p+1, \dots, D$ , upper/lower sign corresponds to untwisted/twisted scalar field, and  $\mathbf{e}_l$  is the unit vector along the direction of the coordinate  $z^l$ .

In order to evaluate the vacuum expectation value (VEV) of the the energy-momentum tensor we will use the direct mode-summation procedure assuming that

the field is prepared in the Bunch-Davies vacuum state. The corresponding eigenfunctions have the form

$$\varphi_\sigma(x) = \left[ \frac{e^{i(\nu-\nu^*)\pi/2}\eta^D}{2^{p+2}\pi^{p-1}V_q\alpha^{D-1}} \right]^{1/2} H_\nu^{(1)}(k\eta) e^{i\mathbf{k}_p \cdot \mathbf{z}_p + i\mathbf{k}_q \cdot \mathbf{z}_q}, \quad \eta = \alpha e^{-t/\alpha}, \quad (4)$$

where  $H_\nu^{(1)}(x)$  is the Hankel function of the order

$$\nu = [D^2/4 - D(D+1)\xi - m^2\alpha^2]^{1/2}, \quad (5)$$

$V_q = L_{p+1} \cdots L_D$  is the volume of the compactified subspace, and

$$\begin{aligned} \mathbf{k}_p &= (k_1, \dots, k_p), \quad \mathbf{k}_q = (k_{p+1}, \dots, k_D), \quad k = \sqrt{\mathbf{k}_p^2 + \mathbf{k}_q^2}, \\ k_l &= 2\pi(n_l + g_l)/L_l, \quad n_l = 0, \pm 1, \pm 2, \dots, \quad l = p+1, \dots, D. \end{aligned} \quad (6)$$

In (6),  $g_l = 0$  for untwisted scalar and  $g_l = 1/2$  for twisted scalar field.

### 3. Vacuum energy-momentum tensor

In this section we investigate the VEV for the energy-momentum tensor of a scalar field in dS<sub>D+1</sub> with toroidally compactified  $q$ -dimensional subspace. This quantity acts as a source of gravity in the semiclassical Einstein equations and plays an important role in modelling self-consistent dynamics involving the gravitational field. Having the complete set of eigenfunctions we can evaluate the vacuum energy-momentum tensor by using the mode-sum formula

$$\langle T_{ik} \rangle_{p,q} = \sum_{\sigma} T_{ik} \{ \varphi_{\sigma}(x), \varphi_{\sigma}^*(x) \}, \quad (7)$$

where the bilinear form  $T_{ik}\{f, g\}$  is determined by the form of the classical energy-momentum tensor for a scalar field. In the problem under consideration the set of quantum numbers  $\sigma$  is specified to  $(\mathbf{k}_p, \mathbf{n}_q)$  with  $\mathbf{n}_q = (n_{p+1}, \dots, n_D)$ . Substituting the eigenfunctions (4) into mode-sum (7) and applying to the series over  $n_{p+1}$  the Abel-Plana summation formula (see, for example, Refs. 3, 10), we find the following recurrence relation for the VEV of the energy-momentum tensor

$$\langle T_i^k \rangle_{p,q} = \langle T_i^k \rangle_{p+1,q-1} + \Delta_{p+1} \langle T_i^k \rangle_{p,q}. \quad (8)$$

Here  $\langle T_i^k \rangle_{p+1,q-1}$  is the part corresponding to dS spacetime with  $p+1$  uncompactified and  $q-1$  toroidally compactified dimensions and  $\Delta_{p+1} \langle T_i^k \rangle_{p,q}$  is induced by the compactness along the  $z^{p+1}$  - direction. For the corresponding energy density one has

$$\begin{aligned} \Delta_{p+1} \langle T_0^0 \rangle_{p,q} &= \frac{2\eta^D L_{p+1}}{(2\pi)^{(p+3)/2} V_q \alpha^{D+1}} \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} \int_0^{\infty} dx \\ &\times \frac{x F_\nu^{(0)}(x\eta)}{(nL_{p+1})^{p-1}} f_{(p-1)/2}(nL_{p+1} \sqrt{x^2 + k_{\mathbf{n}_{q-1}}^2}), \end{aligned} \quad (9)$$

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with the notations  $\mathbf{n}_{q-1} = (n_{p+2}, \dots, n_D)$ ,  $f_\mu(y) = y^\mu K_\mu(y)$ , and

$$F_\nu^{(0)}(y) = y^2 [I'_{-\nu}(y) + I'_\nu(y)] K'_\nu(y) + D(1/2 - 2\xi)y [(I_{-\nu}(y) + I_\nu(y))K_\nu(y)]' + [I_{-\nu}(y) + I_\nu(y)] K_\nu(y) (\nu^2 + 2m^2\alpha^2 - y^2). \quad (10)$$

In Eq. (9), the upper/lower sign corresponds to untwisted/twisted scalar field. The vacuum stresses are presented in the form (no summation over  $i$ )

$$\Delta_{p+1} \langle T_i^i \rangle_{p,q} = A_{p,q} - \frac{4\eta^{D+2} L_{p+1}}{(2\pi)^{(p+3)/2} V_q \alpha^{D+1}} \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} \int_0^\infty dx x K_\nu(x\eta) \times \frac{I_{-\nu}(x\eta) + I_\nu(x\eta)}{(nL_{p+1})^{p+1}} f_p^{(i)}(nL_{p+1} \sqrt{x^2 + k_{\mathbf{n}_{q-1}}^2}), \quad (11)$$

where we have introduced the notations

$$\begin{aligned} f_p^{(i)}(y) &= f_{(p+1)/2}(y), \quad i = 1, \dots, p, \\ f_p^{(p+1)}(y) &= -y^2 f_{(p-1)/2}(y) - p f_{(p+1)/2}(y), \\ f_p^{(i)}(y) &= (nL_{p+1} k_i)^2 f_{(p-1)/2}(y), \quad i = p+2, \dots, D. \end{aligned} \quad (12)$$

The first term on the right of Eq. (11) is given by

$$A_{p,q} = \frac{2\eta^D L_{p+1}}{(2\pi)^{(p+3)/2} V_q \alpha^{D+1}} \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} \int_0^\infty dx \frac{x F_\nu(x\eta)}{(nL_{p+1})^{p-1}} \times f_{(p-1)/2}(nL_{p+1} \sqrt{x^2 + k_{\mathbf{n}_{q-1}}^2}), \quad (13)$$

with the notation

$$F_\nu(y) = [2(D+1)\xi - D/2] y [(I_{-\nu}(y) + I_\nu(y)) K_\nu(y)]' + (4\xi - 1) y^2 K'_\nu(y) \times [I'_{-\nu}(y) + I'_\nu(y)] + [I_{-\nu}(y) + I_\nu(y)] K_\nu(y) [(4\xi - 1)(y^2 + \nu^2)]. \quad (14)$$

As it is seen from the obtained formulae, the topological parts in the VEVs are time-dependent and, hence, the local dS symmetry is broken by them. By taking into account the relation between the conformal and synchronous time coordinates, we see that the VEV of the energy-momentum tensor is a function of the combinations  $L_l/\eta = L_l e^{t/\alpha}/\alpha$ , which is the comoving length of the compactified dimension measured in units of the dS curvature scale.

The recurring application of formula (8) allows us to write the VEV in the form

$$\langle T_i^k \rangle_{p,q} = \langle T_i^k \rangle_{\text{dS}} + \langle T_i^k \rangle_{\text{c}}, \quad \langle T_i^k \rangle_{\text{c}} = \sum_{l=1}^q \Delta_{D-l+1} \langle T_i^k \rangle_{D-l,l}, \quad (15)$$

where the part corresponding to uncompactified dS spacetime,  $\langle T_i^k \rangle_{\text{dS}}$ , is explicitly decomposed. The part  $\langle T_i^k \rangle_{\text{c}}$  is induced by the compactness of the  $q$ -dimensional subspace. This part is finite and the renormalization is needed for the uncompactified dS part only. We could expect this result, since the local geometry is not changed by the toroidal compactification.

For a conformally coupled massless scalar field  $\nu = 1/2$  and, by using the formulae for  $I_{\pm 1/2}(x)$  and  $K_{1/2}(x)$ , after the integration over  $x$  from formulae (9), (11) we find (no summation over  $i$ )

$$\Delta_{p+1}\langle T_i^i \rangle_{p,q} = -\frac{2(\eta/\alpha)^{D+1}}{(2\pi)^{p/2+1}V_{q-1}} \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} \frac{g_p^{(i)}(nL_{p+1}k_{\mathbf{n}_{q-1}})}{(nL_{p+1})^{p+2}}, \quad (16)$$

with the notations

$$\begin{aligned} g_p^{(0)}(y) &= g_p^{(i)}(y) = f_{p/2+1}(y), \quad i = 1, \dots, p, \\ g_p^{(p+1)}(y) &= -(p+1)f_{p/2+1}(y) - y^2 f_{p/2}(y), \\ g_p^{(i)}(y) &= (nL_{p+1}k_i)^2 f_{p/2}(y), \quad i = p+2, \dots, D. \end{aligned} \quad (17)$$

In this case the topological part in the VEV of the energy-momentum tensor is traceless and the trace anomaly is contained in the uncompactified dS part only. Formula (16) could be obtained from the corresponding result in  $(D+1)$ -dimensional Minkowski spacetime with spatial topology  $\mathbb{R}^p \times (\mathbb{S}^1)^q$ , taking into account that two problems are conformally related:  $\Delta_{p+1}\langle T_i^k \rangle_{p,q} = \Delta_{p+1}\langle T_i^k \rangle_{p,q}^{(M)}/a^{D+1}(\eta)$ , where  $a(\eta) = \alpha/\eta$  is the scale factor. This relation is valid for any conformally flat bulk. The similar formula takes place for the total topological part  $\langle T_i^k \rangle_c$ . Note that, in this case the expressions for  $\Delta_{p+1}\langle T_i^k \rangle_{p,q}$  are obtained from the formulae for  $\Delta_{p+1}\langle T_i^k \rangle_{p,q}^{(M)}$  replacing the lengths  $L_l$  of the compactified dimensions by the co-moving lengths  $\alpha L_l/\eta$ ,  $l = p, \dots, D$ .

Now we turn to the investigation of the topological part in the VEV of the energy-momentum tensor in the asymptotic regions of the ratio  $L_{p+1}/\eta$ . For small values of this ratio,  $L_{p+1}/\eta \ll 1$ , to the leading order  $\Delta_{p+1}\langle T_i^k \rangle_{p,q}$  coincides with the corresponding result for a conformally coupled massless field, given by (16). For fixed value of the ratio  $L_{p+1}/\alpha$ , this limit corresponds to  $t \rightarrow -\infty$  and the topological part  $\langle T_i^k \rangle_c$  behaves like  $\exp[-(D+1)t/\alpha]$ . By taking into account that the part  $\langle T_i^k \rangle_{dS}$  is time independent, from here we conclude that in the early stages of the cosmological expansion the topological part dominates in the VEV of the energy-momentum tensor. In particular, in this limit the total energy density is negative.

For small values of the ratio  $\eta/L_{p+1}$ , we introduce a new integration variable  $y = L_{p+1}x$  and expand the integrand by using the formulae for the modified Bessel functions for small arguments. For real values of the parameter  $\nu$  we find

$$\begin{aligned} \Delta_{p+1}\langle T_0^0 \rangle_{p,q} &\approx \frac{2^\nu D [D/2 - \nu + 2\xi(2\nu - D - 1)]}{(2\pi)^{(p+3)/2} L_{p+1}^{-q} V_q \alpha^{D+1}} \Gamma(\nu) \left( \frac{\eta}{L_{p+1}} \right)^{D-2\nu} \\ &\times \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} \frac{f_{(p+1)/2-\nu}(nL_{p+1}k_{\mathbf{n}_{q-1}})}{n^{(p+1)/2-\nu}}. \end{aligned} \quad (18)$$

In particular, this quantity is positive for a minimally coupled scalar field and for a conformally coupled massive scalar field. For a conformally coupled massless scalar

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the coefficient in (18) vanishes. For the vacuum stresses the second term on the right of formula (11) is suppressed with respect to the first term given by (13) by the factor  $(\eta/L_{p+1})^2$  for  $i = 1, \dots, p+1$ , and by the factor  $(\eta k_i)^2$  for  $i = p+2, \dots, D$ . As a result, to the leading order we have the relation (no summation over  $i$ )

$$\Delta_{p+1} \langle T_i^i \rangle_{p,q} \approx (2\nu/D) \Delta_{p+1} \langle T_0^0 \rangle_{p,q}, \quad \eta/L_{p+1} \ll 1, \quad (19)$$

between the energy density and stresses,  $i = 1, \dots, D$ . The coefficient in this relation does not depend on  $p$  and, hence, it takes place for the total topological part of the VEV as well. Hence, in the limit under consideration the topological parts in the vacuum stresses are isotropic. Note that this limit corresponds to late times in terms of synchronous time coordinate  $t$ ,  $(\alpha/L_{p+1})e^{-t/\alpha} \ll 1$ , and the topological part in the VEV is suppressed by the factor  $\exp[-(D-2\nu)t/\alpha]$ . For a conformally coupled massless scalar field the coefficient of the leading term vanishes and the topological parts are suppressed by the factor  $\exp[-(D+1)t/\alpha]$ . As the uncompactified dS part is constant, it dominates the topological part at the late stages of the cosmological evolution.

For small values of the ratio  $\eta/L_{p+1}$  and for purely imaginary  $\nu$ , the energy density behaves like

$$\Delta_{p+1} \langle T_0^0 \rangle_{p,q} \approx \frac{4De^{-Dt/\alpha} B}{(2\pi)^{(p+3)/2} \alpha L_{p+1}^p V_q} \sin[2|\nu|t/\alpha + 2|\nu| \ln(L_{p+1}/\alpha) + \phi_0], \quad (20)$$

where the parameters  $B$  and  $\phi_0$  are defined by the relation

$$Be^{i\phi_0} = 2^{i|\nu|} [|\nu|(1/2 - 2\xi) + i(D/4 - (D+1)\xi)] \Gamma(i|\nu|) \times \sum_{n=1}^{\infty} (\pm 1)^n \sum_{\mathbf{n}_{q-1}=-\infty}^{+\infty} n^{2i|\nu|-p-1} f_{(p+1)/2-i|\nu|}(nL_{p+1} \mathbf{k}_{\mathbf{n}_{q-1}}). \quad (21)$$

In the same limit, the main contribution into the vacuum stresses comes from the term  $A_{p,q}$  in (13) and one has (no summation over  $i$ )

$$\Delta_{p+1} \langle T_i^i \rangle_{p,q} \approx \frac{8|\nu|e^{-Dt/\alpha} B}{(2\pi)^{(p+3)/2} \alpha L_{p+1}^{p+1} V_{q-1}} \cos[2|\nu|t/\alpha + 2|\nu| \ln(L_{p+1}/\alpha) + \phi_0]. \quad (22)$$

Hence, in the case under consideration at late stages of the cosmological evolution the topological part is suppressed by the factor  $\exp(-Dt/\alpha)$  and the damping of the corresponding VEV has an oscillatory nature.

In the special case of topology  $\mathbf{R}^{D-1} \times \mathbf{S}^1$  with the length of the compactified dimension  $L_{p+1} = L$ , for the topological part in the energy density we have

$$\langle T_0^0 \rangle_c = \frac{2(\eta/L)^{D-2}}{(2\pi)^{D/2+1} \alpha^{D+1}} \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^{D-2}} \int_0^{\infty} dx x F_{\nu}^{(0)}(x) f_{D/2-1}(nxL/\eta). \quad (23)$$

We recall that the quantity  $L/\eta$  is the comoving length of the compactified dimension measured in units of the dS curvature scale  $\alpha$ . Note that the corresponding

quantity in the Minkowski spacetime with topology  $\mathbb{R}^{D-1} \times \mathbb{S}^1$  has the form

$$\langle T_0^0 \rangle_c^{(M)} = -\frac{2}{(2\pi)^{(D+1)/2} L^{D+1}} \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^{D+1}} f_{(D+1)/2}(nLm), \quad (24)$$

and is always positive for an untwisted scalar field. In order to illustrate the oscillatory behavior, in Fig. 1 by the full curve we have plotted the topological part in the VEV of the energy density for an untwisted scalar field in  $dS_5$  with topology  $\mathbb{R}^3 \times \mathbb{S}^1$  as a function of the comoving length of the compactified dimension in units of  $\alpha$ :  $L_c = L/\eta$  for the value of the parameter  $\alpha m = 4$ . This topology corresponds to the original Kaluza-Klein model. The dashed curve presents the corresponding quantity in Minkowski spacetime with topology  $\mathbb{R}^3 \times \mathbb{S}^1$  (formula (24) with  $D = 4$ ) as a function of the length of the compactified dimension in the same units:  $L_c = L/\alpha$ .

#### 4. Conclusion

Motivated by the fact that dS spacetime naturally arises in a number of contexts, in the present paper we consider the Casimir densities for a massive scalar field in  $(D+1)$ -dimensional dS spacetime having the spatial topology  $\mathbb{R}^p \times (\mathbb{S}^1)^q$ . Both cases of the periodicity and antiperiodicity conditions along the compactified dimensions are discussed. We have derived a recurrence relation which presents the vacuum energy-momentum tensor for the  $dS_{D+1}$  with topology  $\mathbb{R}^p \times (\mathbb{S}^1)^q$  in the form of the sum of the energy-momentum tensor for the topology  $\mathbb{R}^{p+1} \times (\mathbb{S}^1)^{q-1}$  and the additional part induced by the compactness of the  $(p+1)$ th spatial dimension. The repeated application of the recurrence formula allows us to present the expectation value of the energy-momentum tensor as the sum of the uncompactified dS and

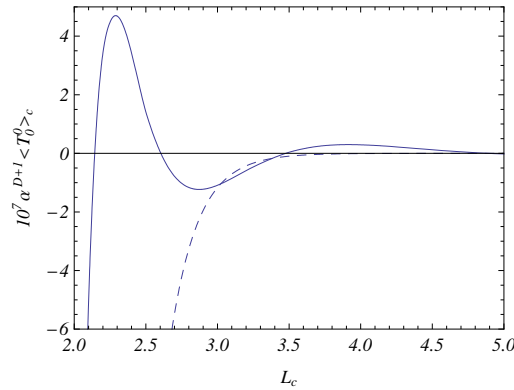


Fig. 1. Topological part of the vacuum energy density,  $\alpha^{D+1} \langle T_0^0 \rangle_c$ , in  $dS_5$  with topology  $\mathbb{R}^3 \times \mathbb{S}^1$  as a function of the comoving length of the compactified dimension in units of  $\alpha$ ,  $L_c = L/\eta$ , for the value of the parameter  $\alpha m = 4$ . The dashed curve presents the corresponding quantity in Minkowski spacetime with topology  $\mathbb{R}^3 \times \mathbb{S}^1$  as a function of the length of the compactified dimension in the same units:  $L_c = L/\alpha$ .

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topological parts. Since the toroidal compactification does not change the local geometry, in this way the renormalization of the energy-momentum tensor is reduced to that for uncompactified  $dS_{D+1}$ .

At early stages of the cosmological expansion, corresponding to  $t \rightarrow -\infty$ , the vacuum energy-momentum tensor coincides with the corresponding quantity for a conformally coupled massless field and the topological part behaves like  $e^{-(D+1)t/\alpha}$ . In this limit the topological part dominates in the VEV. At late stages of the cosmological expansion,  $t \rightarrow +\infty$ , the behavior of the topological part depends on the value of  $\nu$ . For real values of this parameter the leading term in the corresponding asymptotic expansion is given by formula (18) and the vacuum stresses are isotropic. In this limit the topological part is suppressed by the factor  $e^{-(D-2\nu)t/\alpha}$ . In the same limit and for pure imaginary values of the parameter  $\nu$  the asymptotic behavior of the topological part in the VEV of the energy-momentum tensor is described by formulae (20), (22) and the topological terms oscillate with the amplitude going to the zero as  $e^{-Dt/\alpha}$  for  $t \rightarrow +\infty$ .

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